

**Workshop : “Long-Time Behaviour and Statistical Inference for
Stochastic Processes: from Markovian to Long-Memory
Dynamics”
Abstracts**

Program of Wednesday 20/11/2019

C. Tudor: Asymptotic expansion in terms of cumulants.

The recent Stein-Malliavin theory proposes an approach in terms of Malliavin Calculus to estimate the rate of convergence between probability distributions. There exists various methods to estimate the rate of convergence in limit theorems. Certain applications, especially in statistics, need a more deep understanding of such limit theorems, in particular the behavior of the sequence of densities for a sequence of random variables that converges to a Gaussian law. In this talk, we will discuss general criteria based on Malliavin Calculus to find the asymptotic expansion up to a second order term or even to an arbitrary order, of densities. We will explain the role of the cumulants in the asymptotic expansion and we apply our results to the solution to the wave equation with white-noise.

M.J. Garrido-Atienza : Pathwise techniques for rough differential equations: existence and longtime behavior.

In this talk, we will describe two different pathwise approaches for solving rough differential equations. The first one based on compensation of fractional derivatives, whereas the second one rests upon a mixture of rough path theory and fractional calculus. Both methods are therefore useful to establish that stochastic differential equations with general smooth difussions and driven by fractional Brownian motion with $H \in (1/3, 1/2]$ generate random dynamical systems.

S. Lemler: Nonparametric drift estimation for diffusions with jumps driven by a Hawkes process.

We consider a 1-dimensional diffusion process X with jumps. The particularity of this model relies in the jumps which are driven by a multidimensional Hawkes process denoted N . This article is dedicated to the study of a nonparametric estimator of the drift coefficient of this original process. We construct estimators based on discrete observations of the process X in a high frequency framework with a large horizon time and on the observations of the process N . The proposed nonparametric estimator is built from a least squares contrast procedure on subspace spanned by trigonometric basis vectors. We obtain adaptive results that are comparable with the one obtained in the nonparametric regression context. We finally conduct a simulation

study in which we first focus on the implementation of the process and then on showing the good behavior of the estimator.

S. Riedel: A dynamical systems approach to singular stochastic delay differential equations.

Stochastic delay differential equations (SDDE) are prominent examples of stochastic differential equations on infinite-dimensional spaces which, in general, do not generate a stochastic flow. Consequently, a dynamical theory to study these equations seemed to be impossible for a long time. In this talk, we solve this problem by showing that SDDE indeed induce random dynamical systems on an infinite dimensional fiber bundle. On this structure, we can prove a Multiplicative Ergodic Theorem which yields a spectral theory for linear SDDE. As an application, we can prove a stable manifold theorem for singular SDDE. Joint work with Mazyar Ghani Varzaneh and Michael Scheutzow (both TU Berlin).

M. Varvenne: A drift estimation procedure for stochastic differential equations with additive fractional noise

In this talk, we will present some recent joint work with Panloup and Tindel which focuses on the parametric estimation of the drift term in an additive fractional SDE, under ergodic assumptions on the drift coefficient. Our procedure is based on the identification of the invariant distribution for which we build an approximation from discrete-time observations of the SDE. We will give consistency results as well as non-asymptotic estimates of the corresponding quadratic error. We will also discuss about the identifiability condition which is intrinsically linked to the invariant distribution.

Program of Thursday 21/01/2019

F. Comte: Nonparametric drift estimation for i.i.d. paths of stochastic differential equations

We consider N independent stochastic processes $\{(X_i(t), t \in [0, T]), i = 1, \dots, N\}$, defined by a one-dimensional stochastic differential equation, which are continuously observed throughout a time interval $[0, T]$ where T is fixed (large T can be discussed). We study nonparametric estimation of the drift function on a given subset A of \mathbb{R} . Nonparametric projection estimators of the drift function are defined on finite dimensional subsets of $L^2(A, dx)$, and risk bounds are proved, in term of mean integrated quadratic risk. We stress that the set A may be compact or not and the diffusion coefficient may be bounded or not. In a second step, a data-driven procedure to select the dimension of the projection space is proposed, where the dimension is chosen within a random collection of models. Upper bounds on the integrated quadratic risk are also obtained for the final estimator. The results are discussed and simulation results illustrate the method.

F. Merlevède: Rates in almost sure invariance principle for slowly mixing dynamical systems

This talk will concern the one-dimensional almost sure invariance principle with essentially optimal rates for slowly (polynomially) mixing deterministic dynamical systems, such as Pomeau-Manneville intermittent maps, with Hölder continuous observables. Our rates have form $o(n^\gamma L(n))$, where $L(n)$ is a slowly varying function and γ is determined by the speed of mixing. We strongly improve previous results where the best available rates did not exceed $O(n^{1/4})$. To break the $O(n^{1/4})$ barrier, we represent the dynamics as a Young-tower-like Markov chain and adapt the methods of Berkes-Liu-Wu and Cuny-Dedecker-Merlevède on the Komlos-Major-Tusnady approximation for dependent processes. This talk is based on a joint work with C. Cuny, A. Korepanov and J. Dedecker.

C.E. Brehier: Analysis of Adaptive Biasing Methods for diffusion processes

Sampling multimodal distributions μ (in large dimension) is a challenging task. Assume that μ is the ergodic distribution of the overdamped Langevin dynamics $dX_t = -\nabla V(X_t)dt + dW(t)$, then the dynamics is metastable. We will consider the case when the metastable behavior is described by the slow evolution of $\xi(X_t)$. Biasing the dynamics means replacing $V(X_t)$ by $A(\xi(X_t))$. The optimal choice of A is given by a free energy function, which is unknown.

We will introduce adaptive versions of this biasing techniques, where $A = A_t$, which can be seen as diffusions interacting with the past, and prove that they are consistent in the large time limit.

This talk is based on joint works with Michel Benaïm and Pierre Monmarché.

A. Deya: A parabolic Anderson model with space-time fractional noise

The fractional parabolic Anderson model is one of the few fractional SPDEs for which global existence of a solution can be guaranteed, and therefore for which long-time behaviour can be studied.

In this talk, I will present two possible approaches to the problem: the Skorohod interpretation (derived from Malliavin calculus theory) and the pathwise method (based on Hairer's theory of regularity structures). In each case, global existence of a unique solution can be shown under suitable "subcritical" conditions on the fractional noise. Specific "critical" situations can also be exhibited, with a clear finite-time explosion phenomenon.

We will finally see that, to some extent, the Skorohod and pathwise solutions can be compared at the level of their "Feynman-Kac" representations, leading to new asymptotic bounds for the moments of the pathwise solution.

The talk is based on an ongoing work with X. Chen, C. Ouyang and S. Tindel.

N. Berglund: Convergence to equilibrium in some singular parabolic SPDEs

We consider bistable singular parabolic SPDEs of Allen-Cahn type. Unlike in the case of the Φ^4 model, for weak noise, convergence to equilibrium is exponentially slow in these equations. Sharp asymptotics on the speed of convergence, going beyond large-deviation results, can be obtained by potential-theoretic methods. While large-deviation asymptotics are not influenced

by renormalisation counterterms needed to make the SPDEs well-posed, these sharp asymptotics of Eyring-Kramers type do depend on them. In particular, we will see that for the two-dimensional Allen-Cahn equation, the speed of convergence can be expressed in terms of a Carleman-Fredholm determinant. Based on joint works with Ajay Chandra (Imperial College), Barbara Gentz (Bielefeld), Giacomo Di Gesù (Vienna) and Hendrik Weber (Bath).